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Reviewer Name: Sankarnarayanan, Brahadeesh

Mathematical Reviews/MathSciNet Reviewer Number: 169002

Address:
Department of Mathematics
Indian Institute of Technology Jodhpur
NH65, Nagaur Road, Karwar
Jodhpur Rajasthan, 342037
INDIA
briha1994@gmail.com

Author: Jiang, Yiting; Xu, Huijuan; Xu, Xinbo; Zhu, Xuding

Title: Degree-truncated choice number of planar graphs.

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Review text:

All graphs considered are simple and undirected. For a positive integer k, a graph G is said to be *degree-truncated k-choosable* if G is f-choosable for $f(v) = \min\{k, d_G(v)\}$ for each $v \in V(G)$. Note that a connected graph is not degree-choosable if and only if it is a Gallai tree, so such graphs are also not degree-truncated k-choosable for any positive integer k. A connected graph that is not a Gallai tree is degree-truncated Δ-choosable.

Let $\mathcal{P}$ be the class of 3-connected non-complete planar graphs. Richter asked [cf. J. Graph Theory **59** (2008), no. 1, 59–74; MR2432887] whether every graph in $\mathcal{P}$ is degree-truncated 6-choosable. This was answered in the negative by Zhou–Zhu–Zhu [J. Combin. Theory Ser. B **175** (2025), 171–186; MR4933734], who constructed a graph in $\mathcal{P}$ that is not degree-truncated 7-choosable. They also showed that every graph in $\mathcal{P}$ is degree-truncated 16-choosable (in fact, degree-truncated 16-DP-colorable).

In this paper, the authors prove that there exists a graph in $\mathcal{P}$ that is not degree-truncated 8-choosable, and that every graph in $\mathcal{P}$ is degree-truncated 12-choosable. The lower bound is proved by an explicit construction of a 3-connected non-complete planar graph on 1 234 vertices which is not degree-truncated 8-choosable. The authors remark that the proof of the upper bound specifically takes advantage of the list-assignment, and is thus not applicable to DP-colorings, on-line list colorings, or Alon–Tarsi orientations. The authors also announce that the range of k for degree-truncated k-choosability for graphs in $\mathcal{P}$ has been narrowed from $[9, 12]$ to $[10, 11]$ in a subsequent work by H. Xu,

H. Zhou, J. Zhu, and X. Zhu.